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A NOVEL APPROACH TO MEASURING CRITERION WEIGHTS IN MULTIPLE CRITERIA DECISION MAKING: CUBIC EFFECT-BASED MEASUREMENT (CEBM)

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ABSTRACT

In the realm of multi-criteria decision making (MCDM) literature, various approaches exist for quantifying the weight coefficients of criteria. In this study, unlike other methods of calculating weight coefficients, a mathematical model based on cubic interactions among criteria has been proposed (CEBM-Cubic Effect-Based Measurement). This model aims to enrich the MCDM literature while providing a means to compute weight coefficients of criteria. The dataset employed in this investigation comprises criterion values extracted from the Global Innovation Index (GII) evaluations for 19 G20 countries. Through the analysis outcomes, the efficacy of the proposed methodology in objectively deriving criteria weight coefficients for different nations is demonstrated. Furthermore, a comparative analysis is conducted, juxtaposing the proposed method with other objective weighting techniques (ENTROPY, CRITIC, SD, SVP, LOPCOW, and MEREC) as part of a sensitivity, comparison, and simulation analyses. The CEBM method is a credible, reliable and stability objective criterion weighting method, as demonstrated by its sensitivity, comparison, and simulation analyses. The simulation analysis, in particular, showed that the CEBM method is effective in distinguishing the weights of the criteria and is stable across different scenarios. In conclusion, based on all of these evaluations, it is thought that the CEBM method can be used as an objective criterion weighting method and the proposed methodology will make

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substantial contributions to both the domain of cubic functions and the broader MCDM literature.

Keywords: CEBM, cubic function, cubic effect value.

ÇOK KRİTERLİ KARAR VERMEDE KRİTER AĞIRLIKLARINI ÖLÇMEYE YÖNELİK YENİ BİR YAKLAŞIM: KÜBİK ETKİ TABANLI ÖLÇÜM (CEBM)

ÖZ

Çok kriterli karar verme (ÇKKV literatüründe, kriter ağırlık katsayılarını hesaplamak için çeşitli yaklaşımlar mevcuttur. Bu çalışmada, diğer ağırlık katsayıları hesaplama yöntemlerinden farklı olarak, kriterler arasındaki kübik etkileşimlere dayalı bir matematiksel model önerilmiştir (Kübik Etki Tabanlı Ölçüm). Bu model, kriter ağırlık katsayılarını hesaplamak için bir araç sağlarken ÇKKV literatürünü zenginleştirmeyi amaçlamaktadır. Bu araştırmada kullanılan veri seti, 19 G20 ülkesi için Küresel İnovasyon Endeksi (KEİ) değerlendirmelerinden elde edilen kriter değerlerini içermektedir. Analiz sonuçları, önerilen metodolojinin farklı ülkeler için kriter ağırlık katsayılarını objektif olarak türetmede etkinliğini göstermektedir. Ayrıca, önerilen yöntem ile diğer objektif ağırlıklandırma yöntemleri (ENTROPY, CRITIC, SD, SVP, LOPCOW ve MEREC) arasında duyarlılık, karşılaştırma ve simülasyon analizleri kapsamında karşılaştırmalı bir analiz yapılmıştır. CEBM yöntemi, duyarlılık, karşılaştırma ve simülasyon analizleri ile gösterildiği üzere, güvenilir ve kararlı bir objektif kriter ağırlıklandırma yöntemidir. Özellikle simülasyon analizi, CEBM yönteminin kriterlerin ağırlıklarını ayırt etmede etkili olduğunu ve farklı senaryolarda kararlı olduğunu göstermiştir. Sonuç olarak, tüm bu değerlendirmelere göre CEBM yönteminin objektif bir kriter ağırlıklandırma yöntemi olarak kullanılabileceği ve önerilen metodolojinin hem kübik fonksiyonlar alanına, hem de ÇKKV literatürüne önemli katkılar sağlayacağı düşünülmektedir.

Anahtar Kelimeler: CEBM, kübik fonksiyon, kübik etki değeri.

1. INTRODUCTION

In the context of multi-criteria decision-making (MCDM) involving various criteria and distinct decision options, a pivotal aspect is the assessment of significance levels or weight coefficients assigned to the criteria. Given that each criterion bears a distinct weight, the arrangement of decision alternatives is inherently influenced by the significance attributed to these criteria. Hence, the process of attributing weightages to the criteria in alignment with their impact on decision alternatives assumes paramount importance in the practical implementation of the MCDM technique. To elucidate further, criterion weighting denotes a procedure wherein distinct weights are allocated to each criterion to accurately mirror their relative significance within the decision-making process. These weightings are contingent upon various approaches, encompassing expert evaluations, surveys, or statistical analyses. Once the criterion weights are ascertained, they serve as the foundation for appraising decision alternatives based on their comprehensive performance. Undoubtedly, the task of criterion weighting constitutes a pivotal stride within the MCDM process. Its essence lies in ensuring equitability and objectivity throughout decision-making, thereby enabling an authentic valuation of decision alternatives.

Within the realm of MCDM literature, a plethora of techniques exists for quantifying weight coefficients. In tandem with these established methodologies, the current study has pioneered a novel approach to compute objective weight coefficients for variables. This innovative technique operates within the framework of cubic relationships between variables, utilizing cubic functions. The primary thrust of this study resides in the scrutiny and harnessing of cubic functions' analytical and modeling prowess in determining weight coefficients. Cubic functions, renowned for their efficacy in resolving diverse problems across various domains, occupy a pivotal role in this investigation. In addition, in the MCDM literature, there are limited methods that explain the weight coefficients of criteria by basing them on nonlinear functions between criteria and the interaction structure between criteria. Therefore, the other important motivation for the development of the method is the ability to provide policies for the improvement of specific criteria or criteria through the interaction structure between criteria that can be detected by the CEBM method in complex problems.

Consequently, the study's objectives unfold in two facets. Firstly, it endeavors to introduce an innovative paradigm for assessing weight coefficients concerning decision alternatives in the domain of MCDM. Secondly, it aims to kindle a nuanced comprehension of the capabilities inherent to cubic functions, recognizing their aptitude in dissecting and resolving intricate predicaments. To this end, the literature review segment of the study expounds upon objective weighting methodologies and delves into the mechanics of cubic functions. The subsequent section outlining the methodology delineates the research dataset and articulates the proposed approach. In the results segment, discerning observations are drawn and meticulously deliberated, encapsulating the quantitative outcomes gleaned within the purview of the study's ambit.

2. LITERATUR REVIEW

When making decisions, it is important to consider the relative importance of different criteria. This is because different alternatives may perform differently on different criteria, and it is necessary to compare their overall performance in order to make the best decision (Saaty, 1997).

Historically, the assessment of criteria significance has relied on weight coefficients, which can be ascertained either through subjective or objective means. Subjective weight coefficients are contingent upon the evaluator's personal experiences and judgments, whereas objective weight coefficients are derived through mathematical models. Subjective weight coefficients are frequently gleaned from the insights of experts in the field. Nonetheless, it's imperative to acknowledge that expert perspectives might carry inherent biases, thereby introducing potential inaccuracies into the decision-making process due to the subjective nature of these evaluations. In contrast, objective weight coefficients remain impervious to the decision-maker's predispositions or uncertainties. Consequently, these coefficients are generally regarded as more precise in comparison to their subjective counterparts (Arslan, 2020; Bardakçı, 2020: 20; Demir, 2020), as they are grounded in empirical analysis and remain insulated from personal inclinations.

In summary, the precise determination of criteria's relative significance stands as a pivotal stride within the decision-making process. Both subjective and objective weight coefficients serve as tools to gauge the relative importance of criteria. However, it is widely acknowledged that objective weight coefficients generally yield a higher degree of accuracy compared to their subjective counterparts. The literature on MCDM showcases an array of objective weighting methodologies. Among these techniques are ENTROPY, CRITIC (Criteria Importance Through Intercriteria Correlation), CILOS (Criterion Impact Loss), IDOCRIW (Integrated Determination of Objective Criteria Weights), SD (Standard

Deviation), SVP (Statistical Variance Procedure), SECA (Simultaneous Evaluation of Criteria and Alternatives), MEREC (Method Based On Removal Effects of Criteria), and LOPCOW (Logarithmic Percentage Change-driven Objective Weighting).

The ENTROPY method is based on the concept of entropy, which measures the disorder or uncertainty of a system. In this sense, the more disorder a criterion has, the more distinct it will be from others and become the most important criterion. Therefore, the ENTROPY method can be effectively used in the decision-making proces In this method, after preparing the decision matrix, the standard values of the decision matrix and the entropy measurement of the criteria are used to determine the entropy weights of the criteria. The entropy weights are calculated as the inverse of the entropy value The entropy weights are a measure of the relative importance of the criteria (Ayçin, 2019).

The CRITIC method is a MCDM approach designed to derive criterion weights through an examination of their interrelationships. This method commences by constructing a decision matrix, which delineates the performance of various decision alternatives across distinct criteria. Subsequently, the decision matrix values undergo normalization, facilitating their transformation into a unified scale within the 0 to 1 range. The ensuing step involves an analysis of the criterion relationships, predicated on the normalized values. This analytical process is instrumental in identifying any inconsistencies or contradictions that may arise between criteria. The resolution of these contradictions is executed by leveraging the concept of standard deviation as a weighting mechanism. The ultimate outcome of the CRITIC method is the computation of criterion weights, which are inversely proportional to the identified contradictions. This configuration ensures that criteria with higher contradictions hold diminished weightage, aligning with the endeavor to achieve a coherent and balanced decision-making framework (Diakoulaki, Mavrotas and Papayannakis, 1995).

The CILOS method constitutes a MCDM approach designed to ascertain criterion weights grounded in the variance between other criteria's ideal maximum and minimum values. The initial step of this method entails the computation of a decision matrix, a tabulation that expounds the performance of decision alternatives across diverse criteria. Subsequently, the values within the decision matrix are subjected to normalization, facilitating their transformation into a standardized range spanning from 0 to 1. Subsequent to normalization, a square matrix is constructed, capturing the influence each criterion wields upon the remaining criteria. This step quantifies the impact of each criterion within the context of the others. Subsequently, a weight system matrix is formulated, shedding light on the relative significance of individual criteria within the overarching decision framework. The crux of the CILOS method culminates in the determination of criterion weight coefficients. This is achieved through the resolution of a system of linear equations, ensuring that the resulting weights encapsulate the intricate relationships between the criteria (Zavadskas and Podvezko, 2016; Sel, 2020).

The IDOCRIW method presents a hybridized approach within the domain of MCDM, amalgamating the principles of both the ENTROPY and CILOS methodologies. The central tenet of this method revolves around the assessment of relative impact in the context of an absent index. To outline its operational procedure, the IDOCRIW method initially computes criterion weights utilizing the ENTROPY and CILOS techniques, drawing upon the values inherent in the decision matrix. The resultant ENTROPY and CILOS weights are subsequently amalgamated, yielding the comprehensive IDOCRIW weights, which encapsulate the intricacies of both methodologies (Zavadskas and Podvezko, 2016; Ecer, 2020).

The concept of standard deviation (SD) finds its application as a statistical metric that gauges the extent of dispersion among values within probability and statistics. An alternative definition characterizes it as the square root of variance – the arithmetic mean of the squared disparities between the mean and individual data points.

In the SD methodology, the determination of criterion importance levels or weight coefficients hinges on assessing the normalized values of the criteria. This approach takes into account the significance of scale variation in computing the weight coefficients, acknowledging its role in the process (Demir et al. 2021). The SD method facilitates the objective calculation of criterion importance degrees, relying on the standard deviation value attributed to each criterion (Diakoulaki et al. 1995). Operationalizing the SD method involves straightforward mathematical operations and is devoid of criterion-specific constraints (Wang, 2003).

SVP stands as a target weighting technique aimed at generating objective weights for the computation of criterion weights or significance levels (Nassar, 2019; Demir, Özyalçın and Bircan, 2021). Within this method, the weight values assigned to criteria undergo an objective quantification, thereby remaining impervious to the influence of expert viewpoints and subjective assessments. Furthermore, the method's approach to calculating criterion weights hinges on the variance metrics attributed to the criteria (Gülençer and Türkoğlu, 2020). Upon determining the variance values associated with the criteria, the weights for each criterion are computed by dividing the individual criterion's variance value by the aggregate variance value encompassing all criteria. In essence, the SVP method emerges as an objective weight determination technique, facilitating the calculation of criterion weights or importance levels through the utilization of variance values attributed to the criteria (Odu, 2019).

The SECA technique made its entry into the realm of Multi-Criteria Decision Making (MCDM) literature in 2018, introduced by Keshavarz-Ghorabaee et al. (2018). This method possesses a distinctive attribute that sets it apart from other MCDM techniques: its capability to simultaneously ascertain both criterion weight values and decision alternative performance in relation to those criteria. This distinctive feature renders the SECA method unique within the landscape of MCDM methodologies (Keshavarz-Ghorabaee et al. 2018). The initial stride of this method encompasses the creation of a decision matrix. Subsequently, the values within this matrix undergo a process of standardization. The third stage involves the identification of conflict degrees, followed by the determination of standard deviation values in the fourth step. Moving forward, the fifth step computes standardized values by amalgamating the results derived from the standard deviation and relationship assessments. The conclusive stage involves the solution of a multi-objective linear model (Keshavarz-Ghorabaee et al. 2019: 190-191). This model comprises three distinct objective functions. The foremost objective aims to maximize the scores of decision alternatives, while the second and third objectives focus on minimizing both intra-criteria and inter-criteria deviations. In this intricate framework, the model seeks to minimize the disparity among criterion weight reference points, thereby ensuring the elevation of each decision alternative's performance to its utmost potential (Ecer, 2020).

The MEREC method, classified under the multi-criteria decision analysis (MCDA) umbrella, serves as a mechanism to deduce criterion weights. The method unfolds through a sequence of steps, commencing with the creation of a decision matrix – a tabular representation depicting the performance of decision alternatives across distinct criteria. Subsequent to this, a normalized decision matrix emerges, delineating the performance of decision alternatives across criteria while being scaled within the range of 0 to 1. Moving forward, the total performance values of decision alternatives are computed utilizing a structure rooted in natural logarithms. This entails the summation of natural logarithms of a decision alternative's values across all criteria. Subsequently, changes in performance values of other decision alternatives are evaluated, employing the natural logarithmic approach. This pertains to calculating the discrepancy between a decision alternative's performance value on a specific criterion and its corresponding value in the normalized decision matrix. The culmination of the MEREC method is the determination of criterion weight values, hinging on the calculation of subtraction effects or the sum of absolute deviations. This procedure entails the computation of a criterion's weight value as the summation of absolute values of changes in performance values of other decision alternatives corresponding to that criterion. Although relatively nascent, the MEREC method has demonstrated efficacy across diverse applications. It proves particularly adept in scenarios characterized by interrelated criteria, aiming to minimize disparities between the most and least significant criteria (Keshavarz-Ghorabaee et al. 2021).

The LOPCOW method, an acronym for Logarithmic Percentage Change-driven Objective Weighting, emerges as an objective weight determination approach that amalgamates information spanning distinct dimensions to derive fitting or ideal weights. This method is further designed to attenuate disparities between criteria of varying importance levels, while concurrently acknowledging the interconnectedness between criteria. Initiating its course, the LOPCOW method prepares a decision matrix, a tabulated representation elucidating the performance of decision alternatives concerning diverse criteria. Subsequently, values within this decision matrix undergo normalization, thereby standardizing values within a range of 0 to 1. Furthering the procedure, the calculation of the average square value, expressed as a percentage of the criterion's standard deviation, takes place. This computation serves to mitigate discrepancies (gaps) arising due to the scale of data. The subsequent derivation of weight coefficients for criteria rests on the inverse of this average square value. Although relatively recent, the LOPCOW method has demonstrated its effectiveness across an array of applications. It particularly excels in contexts marked by criterion interdependencies, where bridging gaps between highly and less significant criteria assumes paramount importance (Ecer and Pamucar, 2022).

Within the domain of MCDM literature, the objective weights assigned to criteria unveil two pivotal characteristics. The first hallmark centers on the disparity in performance exhibited by decision alternatives across each criterion. This quantifiable measure denotes the divergence between the highest and lowest values among criteria. The second characteristic pertains to the individuation or contention prevailing among criteria. This dimension encapsulates the extent to which criteria diverge from one another. By harnessing and leveraging these inherent characteristics, which lie embedded within the data characterizing a multi-criteria problem, decision-makers stand to gain substantial insights within the decision-making process. For instance, when the objective weights of criteria underscore a heightened degree of contention among them, it may prompt decision-makers to accord priority to certain criteria or opt for an alternative decision-making methodology. These dual characteristics of objective criterion weights wield profound significance for decision-makers throughout the MCDA process. The comprehension of these characteristics equips decision-makers to formulate improved choices that harmonize more effectively with their objectives (Ecer, 2020).

Apart from the aforementioned attributes, criteria also possess the potential to interrelate in terms of their quantitative outcomes. This interplay can manifest as one criterion exerting an impact on another. For instance, if one criterion positively influences another, strategic measures can be devised to enhance the influenced criterion's performance. Conversely, if a positive influence leads to a decline in the development of the influenced criterion, strategies can be implemented to mitigate or curtail the influencing criterion's effect on the influenced one. In accordance with this rationale, avenues emerge for devising strategies, policies, and recommendations that facilitate the advancement of criteria through the lens of interrelationships among criteria within any given concept. In this context, the application of cubic functions becomes pertinent for gauging criterion weight coefficients. This stems from the fact that cubic functions facilitate the determination of values wherein criteria mutually influence one another, functioning as dependent and independent variables. Thus, cubic functions offer a methodology to ascertain these interdependent values among influencing and influenced criteria (Karagöz, 2017).

The literature underscores several advantageous aspects associated with cubic functions. Firstly, cubic functions provide a versatile means to effectively model real-world relationships between variables. This attribute proves particularly invaluable in nonlinear modeling scenarios, where the flexibility inherent in cubic functions contributes to the model's meaningfulness and constructiveness. Secondly, cubic functions demonstrate an ability to mitigate overfitting more effectively than higher-degree polynomials. This quality engenders greater consistency in the relationship between variables, enhancing the robustness of the model. Thirdly, cubic functions inherently possess a maximum of three real roots, invariably situated at the function's zero points. This inherent characteristic guarantees the existence of at least one local minimum or maximum point within the function. Consequently, this distinctive trait can be harnessed to optimize the cubic relationship between variables.

The convergence of these benefits has rendered cubic functions a staple in diverse applications spanning fields such as physics, chemistry, and economics. Instances of their application include the modeling of object motion, molecular structures, and economic growth. Collectively, cubic functions manifest as a potent tool for capturing intricate relationships between variables. Their significance is particularly pronounced in nonlinear modeling endeavors, where they imbue the model with enhanced meaningfulness, consistency, and optimizability (Abramowitz and Stegun, 1965; Neumark, 1965).

The foundation of cubic functions is rooted in polynomial function. linear function, given by $f(x) = mx + b$ where $(m \neq 0)$, is a polynomial function of degree 1. A quadratic function, expressed as $f(x) = ax^2 + bx + c$ where $(a \neq 0)$, falls within the scope of polynomials with degree 2. Consequently, polynomial functions are constructed in the form of $f(x) = a_n x^n + a_{n-1} x^{n-1} \dots \dots + a_1 x + a_0$, where n is a non-negative integer denoting the degree of the polynomial. The value of n in the equation signifies the polynomial's degree. Additionally, the coefficients $a_0, a_1, \dots \dots a_n$ in the equation are real numbers, and$(a \neq 0)$ is explained as non-zero (Barnett, Ziegler and Byleen, 2015). Likewise, the equation $f(x) = ax^3 + bx^2 + cx + d$ is classified as a cubic function since it is of the third degree (Thomas et al. 2009: 30).

Upon reviewing the existing literature, it becomes evident that a substantial body of research delves into the realm of cubic functions. In this context, Wanninkhof and McGillis (1999) embarked on an exploration into the plausibility of a cubic correlation between gas exchange and instantaneous (or short-term) wind speed. Their investigation encompassed both laboratory and field findings, as well as an assessment of the potential ramifications of this correlation on global air-sea fluxes. The authors articulated that the underpinning theory of this correlation revolves around the retardation induced by surfactants in conditions of low and moderate winds, coupled with bubble-facilitated transfer under high-wind conditions. Notably, the authors observed that the cubic correlation they proposed, in contrast to preceding associations, signifies a subdued gas transfer at low wind speeds and markedly heightened gas transfer at elevated wind speeds. Their conclusion pointed towards the cubic relationship as a more precise representation of the interplay between gas exchange and wind speed, surpassing earlier formulations. This elucidated correlation holds the potential to substantially enhance our comprehension of global air-sea fluxes.

Landquist et al. (2010) studied a survey of cubic function fields with at least fifth character. In their research, they described a technique for defining the signature of any rational place in a cubic extension and pointed out the role of signature calculation in calculating the class number of the function field. Therefore, in their research, the theory of cubic function fields and the study of the zeros of the zeta functions of function fields were evaluated from a different and original perspective.

Gilkar and Sahdad (2014) conducted a study that underscores the efficacy of incorporating cubic functions to enhance the performance of the congestion control mechanism within extensive and expansive networks. The authors additionally proffered an algorithm integrated into the Linux operating system, designed to convert the congestion window into a cubic function. The congestion control mechanism plays a pivotal role in ensuring the network transmits data at a pace commensurate with its capacity. This is achieved through dynamic adjustments in the size of the data window allowed for transmission at any given juncture. Notably, this window size expands during non-congested periods and contracts during periods of network congestion. It was determined by the authors that the integration of cubic functions substantiates an enhancement in the responsiveness of the congestion control mechanism to shifts in network conditions. This enhancement stems from the superior ability of cubic functions to emulate network behavior with heightened precision compared to alternative function types, such as linear functions. Moreover, the algorithm formulated by the authors is straightforward to implement and possesses applicability across diverse network scenarios. Its utilization holds the potential to markedly enhance the efficiency of the congestion control mechanism, particularly within expansive and extensive networks.

Rashid et al. (2018) embarked on the development of cubic functions, utilizing them to construct cubic line graphs, cubic hypergraphs, and cubic soft graphs. The study elucidated that by adopting an alternative perspective, cubic function graphs can be effectively classified. Demonstrating the diverse utility of cubic functions, the authors showcased their efficacy in generating an array of graphs tailored for visualizing various phenomena. The significance of their work lies in its potential to democratize the understanding of cubic functions, rendering them more accessible to a broader audience. Simultaneously, this endeavor serves to foster the adoption of cubic functions across a multitude of disciplines and domains.

Li et al. (2019) extended the expansion elements of the Taylor series to the third order through cubic functions. The study commenced by conducting an analysis of quality loss coefficients, subsequently furnishing a cubic quality loss function. Furthermore, the study introduced a methodology for calculating hidden quality costs employing the cubic loss function. The study's findings underscore the applicability of the cubic quality loss function in quality cost calculation, while indicating that the quadratic loss function is unsuitable for this purpose.

In a parallel study, Muhiuddin et al. (2020) delved into the exploration of cubic functions to ascertain the equivalent condition for cubic inflection points. This endeavor was achieved through elucidating concepts such as cubic path, cubic cycle, cubic diameter, complete cubic graph, and strong cubic. As a result, the authors derived insights that allowed them to utilize cubic graphs in traffic flow scenarios, thereby minimizing the time required to reach destinations.

Tiruneh et al. (2020) presented a method for solving cubic equations that only requires function evaluation. The authors argued that their method eliminates the need to manipulate the original coefficients of the cubic polynomial, and as a result, the solution of cubic equations is easier and more understandable. Additionally, the authors showed that their method can be used to indirectly calculate the roots of a cubic polynomial by using the values of the polynomial at a single point. Therefore, it is considered that the method could simplify the reduction of cubic values, simplify the solution of cubic equations, and make cubic functions more useful in practical application.

Zahedi et al. (2022) underscored the extensive utilization of cubic functions within the realm of engineering. The authors expounded upon the congruence between cubic functions and pertinent parameters, including real gas properties, the degree of chemical equilibrium, and the actual beam deflection. Furthermore, the authors substantiated that cubic equations can be effectively solved through the Cordano formula and the Newton-Raphson method.

3. METHOD

3.1. Data Set and Analysis of the Study

The data set of the study consisted of values of the Global Innovation Index (GII) criteria for 19 countries in the G20 group for 2022. Furthermore, all GII criteria have been determined in a benefit-oriented manner. In the study, the weight coefficients of the GII criteria were calculated using the proposed method. For the convenience of the study, the GII abbreviations are explained in Table 1.

Table 1. GII Criteria and Abbrevations

Criteria	Abbrevations
Institutions	GII1
Human capital and research	GII2
Infrastructure	GII3
Market sophistication	GII4
Business sophistication	GII5
Knowledge and technology outputs	GII6
Creative outputs	GII7

3.2. Proposed Method: Cubic Effect Based Measurement (CEBM)

In the context of two interrelated variables, their quantitative interactions can be elucidated through a range of functions. Within the SPSS literature, these functions encompass linear, quadratic, compound, growth, logarithmic, cubic, S-shaped, exponential, inverse, power, and logistic forms. Depending on the nature of these functions, the reciprocal influences of the variables, whether as dependent or independent, are articulated through equations facilitated by the SPSS program's Curve Estimation feature (Karagöz, 2020: 844-845).

Cubic functions encompass a broader spectrum of data compared to numerous other functions. Moreover, the inherent flexibility of cubic functions allows for the construction of intricate models grounded in empirical data. This adaptability lends itself to the accurate quantification of intervariable effects through cubic functions (Sullivan, 2014). Consequently, within the framework of the proposed methodology, the interactions among criteria were assessed utilizing cubic functions.

When crafting a cubic function between two variables via the SPSS program's Curve Estimation, it becomes feasible to compute the alteration in the dependent variable arising from shifts in the independent variable across the data set's maximum and minimum values. This calculation can be achieved through a specific integral. Essentially, this implies that the alterations in the independent variable provoke or impact the overall variation observed in the dependent variable.

The indefinite deriviate of the function $f(x)$ is denoted by $f'(x)$. Since $f'(x) = \frac{df(x)}{dx}$, $f'(x)dx = df(x)$ can be written. This expression is written as $\int f'(x)dx = \int df(x)$ with the integral sign $\int$, which is the symbol of infinite and continuous sum. The equation

$\int f'(x)dx = f(x)$ can be obtained from this equation. Therefore, the function whose integral is to be found is $f'(x)$. Next, if $\int f(x)dx = F(x) + C$, $\int_r^p f(x)dx = F(p) - F(r)$ is written. Here, 'r' represents the lower limit of the integral, and 'p' represents the upper limit (Kartal, Karagöz and Kartal, 2014). Therefore, after determining the cubic relationships between the criteria with the logarithm function $(y = ax^3 + bx^2 + cx + d)$ the change in the x independent variable between the 'p' and 'r' limits can be measured or affected by the 'y' variable with the definite integral. The steps of applying the proposed method are explained below.

Step 1: Obtaining the Decision Matrix

i: 1, 2, 3...n, where n represents the number of decision alternatives

j: 1, 2, 3,...m, where m represents the number of criteria

D: Decision matrix

C: Criterion

d_{ij}: The decision matrix is constructed according to Equation 1, where "i_j" represents the i-th decision alternative on the j-th criterion.

$$D = \left[d_{ij}\right]_{nxm} = \begin{bmatrix} C_1 & C_2 & \dots & C_m \\ x_{11} & x_{12} & & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \quad (1)$$

Step 2: Normalization of Decision Matrix (d_{ij}^*)

The normalization of the decision matrix is conducted through the utilization of the subsequent equation. Benefit criteria undergo normalization using Equation 2, whereas cost criteria are subjected to normalization employing Equation 3.

$$d_{ij}^* = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}} (2)$$

$$d_{ij}^* = \frac{x_j^{max} - x_{ij}}{x_j^{max} - x_j^{min}} (2)$$

Step 3: Generation of Cubic Functions

Based on the number of criteria, m, cubic functions $(y = ax^3 + bx^2 + cx + d)$ are generated for the variables up to a quantity of $\left\{2.C(m,2) = 2.\frac{m!}{2!.(m-2)!}\right\}$ using SPSS assistance (CURVE ESTIMATION), considering the cubic relationship between them.

$$(1) f(C_1) = C_2, f(C_1) = C_3, \dots\dots f(C_1) = C_m (4)$$

$$(2) f(C_2) = C_1, f(C_2) = C_3, \dots\dots f(C_2) = C_m (5)$$

$$(3) f(C_3) = C_1, f(C_3) = C_2, \dots\dots f(C_3) = C_m (6)$$

$$\vdots \quad \vdots \quad \vdots\vdots \quad \vdots \quad \vdots\vdots \quad \vdots \quad \vdots$$

$$(m) f(C_m) = C_1, f(C_m) = C_2, \dots\dots f(C_m) = C_{m-1} (7)$$

Step 4: Calculation of Cubic Impact Value between Criteria

In this step, the extent to which an independent variable (one criterion) influences or changes a dependent variable (another criterion) is determined by evaluating the independent variable's effect within the range of its maximum and minimum values using definite integral calculation. Here, k represents the cubic impact value of one criterion on the other. It is important to ensure the absolute value of the impact values after the integral calculation.

$$(1) f(C_1) = C_2, \int_{C_{1min.}}^{C_{1maks.}} (f'(C_1))dx = \left|k_{C_1 \to C_2}\right| (8)$$

$$(2) f(C_1) = C_3, \int_{C_{1min.}}^{C_{1maks.}} (f'(C_1))dx = \left|k_{C_1 \to C_3}\right| (9)$$

$$(3) f(C_1) = C_4, \int_{C_{1min.}}^{C_{1maks.}} (f'(C_1))dx = \left|k_{C_1 \to C_4}\right| (10)$$

$$\vdots \quad \vdots \quad \vdots\vdots \quad \vdots \quad \vdots\vdots \quad \vdots \quad \vdots$$

$$\left(\frac{m!}{(m-2)!}\right) f(C_m) = C_{m-1}, \int_{C_{mmin.}}^{C_{mmaks.}} (f'(C_1))dx = \left|k_{C_m \to C_{m-1}}\right| (11)$$

The absolute value of the impact value of one criterion on another criterion is emphasized above. This is because in this method, what matters is not the direction of the influence between criteria, but rather the magnitude of the influence.

Step 5: Calculation of the Total Cubic Impact Values of Each Criterion (T_c)

In this step, the cubic impact values of a criterion on other criteria are summed to measure the overall cubic impact value of a criterion on the other criteria.

$$(1) for C_1 \left|k_{C_1 \to C_2}\right| + \left|k_{C_1 \to C_3}\right| + \left|k_{C_1 \to C_4}\right| \dots \dots + \left|k_{C_1 \to C_m}\right| = \left(\sum_{j=1}^{m-1} \left|k_{C_1 \to C_{j+1}}\right| \right) = T_{C_1} (12)$$

$$(2) for C_2 \left|k_{C_2 \to C_1}\right| + \left|k_{C_2 \to C_3}\right| + \left|k_{C_2 \to C_4}\right| \dots \dots + \left|k_{C_2 \to C_m}\right| = \left(\sum_{j=0, j \neq 1}^{m-1} \left|k_{C_2 \to C_{j+1}}\right| \right) = T_{C_2} \ (13)$$

$$(3) for C_3 \left|k_{C_3 \to C_1}\right| + \left|k_{C_3 \to C_2}\right| + \left|k_{C_3 \to C_4}\right| \dots \dots + \left|k_{C_3 \to C_m}\right| = \left(\sum_{j=0, j \neq 2}^{m-1} \left|k_{C_3 \to C_{j+1}}\right| \right) = T_{C_3} (14)$$

$$\begin{array}{ccccccc} \vdots & \vdots & \vdots\vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots\vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots\vdots & \vdots & \vdots\vdots & \vdots & \vdots \end{array}$$

$$(m) for C_m \left|k_{C_m \to C_1}\right| + \left|k_{C_m \to C_2}\right| + \left|k_{C_m \to C_3}\right| \dots \dots + \left|k_{C_m \to C_{m-1}}\right| = \left(\sum_{j=1}^{m-1} \left|k_{C_m \to C_j}\right| \right) = T_{C_m} (15)$$

Step 6: Determination of Criterion Weight Values $(w| \quad |j)$

In this step, the total cubic impact value of each criterion on the other criteria is divided by the sum of the total cubic impact values of all criteria. This allows for the calculation of the weight coefficient of each criterion.

$$w_j = \frac{T_{C_j}}{\sum_{j=1}^{m} T_{C_j}} (16)$$

The advantages of the CEBM method can be classified into two categories: quantity and quality. These advantages are explained below in bullet points:

Quantity-Based Advantages of the CEBM Method:

Capturing Interactions Among Criteria with Higher Sensitivity: The method's ability to model interactions among criteria using cubic functions enables a more precise capture. Thus, this circumstance can assist in reflecting intercriteria interactions in complex decision-making problems more effectively in the real world. Because with the cubic approach, more realistic results can be obtained compared to the linear approach.

Determination of Criterion Significances: Through the method, criterion weights can be quantitatively calculated, allowing for the measurement of the actual impacts of each criterion on the decision. In this regard, the method reflects the true value of criteria in the decision-making process, thereby establishing a more robust foundation.

Data-Driven Approach: The method contributes to a decision-making process that is objective and data-driven.

Suitability for Complex Decision-Making Problems: The method is specifically designed for use in complex decision-making problem. In this context, cubic functions in the method better capture intricate interactions among criteria and yield more realistic outcome.

Interaction Flexibility: The incorporation of cubic functions introduces a notable degree of flexibility when delineating interactions among criteria. This newfound flexibility proves invaluable in accurately modeling relationships among criteria, particularly within intricate decision-making scenarios. Cubic functions afford the capacity to represent the influences of criteria on one another in a nonlinear fashion. Consequently, this capability facilitates the capture of more intricate and lifelike interactions among criteria, surpassing the confines of mere linear associations. This facet gains utmost importance as it allows for the faithful portrayal of complex decisions and the attainment of heightened outcomes.

Being sensitive to values within the range of [-1, 0] in the decision matrix and during the normalization processes: Some criterion (ENTROPY and MEREC) weighting methods have mathematical limitations that make it difficult to calculate criterion weights when decision alternatives have values in the range of [-1, 0] in the decision and normalization matrices. Consequently, under such circumstances, various transformations are applied to the values within the decision matrix to facilitate the progression of methodological steps for determining criterion weights (e.g., employing Z-scores). These transformations are particularly crucial for methodologies such as ENTROPY and MEREC, as they involve the utilization of logarithmic calculation techniques. However, within the framework of the CEBM method, which relies on cubic functions for computations, no such transformations are requisite for values within the [-1, 0] range within the decision and normalization matrices. As a result, the steps of the CEBM method navigate without encountering undefined values, and the original values within the decision matrix are taken into consideration. In the CRITIC method, the weight coefficient of a criterion increases as the positive directional relationships between the criteria decrease and the standard deviation values increase. In the method, the relationships between the criteria are used to calculate the Pearson correlation coefficient, and the relationships in question take on a linear structure. However, in the CEBM method, the relationships are not in a linear structure, so it is thought that its sensitivity to complex problems is higher than the CRITIC method. In the SD, SVP and LOPCOW methods, the interaction structure between the criteria is not taken into account. However, in the CEBM method, the complex interaction structure between the criteria is taken into account, and the weight value of the criterion with the highest impact value is the highest, as in the DEMATEL method. Therefore, provided that the relationships between the criteria have a theoretical structure, the CEBM method can provide policies for the improvement and development of criteria by taking into account the interaction structure between the criteria.

Qualitature-Based Advantages of the CEBM Method:

Identification of Enhancement Opportunities: Criteria endowed with greater weightage wield amplified influence over other criteria, rendering them apt targets for discerning avenues of enhancement. Grasping the interrelationships among criteria and ascertaining their impact values delineates the domains where endeavors for enhancement should be channeled. Put differently, it engenders the capacity to delve into theoretical causal relationships between criteria, thereby ascertaining the trajectory of their influence. This, in turn, streamlines the process of identifying which criteria warrant prioritization or refinement contingent upon the array of decision alternative

Priority Ascertainment: The process of weight determination facilitates the recognition of priority hierarchies among the criteria. Criteria endowed with higher weights assume a more significant role in comparison to their counterparts. This enables the identification of criteria deserving enhanced attention during strategic planning and the decision-making process.

Performance Appraisal: The coefficients of weightage can be judiciously harnessed for evaluating the performance of the criteria. Criteria endowed with elevated weights are accorded the status of wielding more pronounced influence upon the operational performance of the organization or system. This, in turn, facilitates more efficacious performance assessment and focused endeavors for enhancement, honing in on the pivotal criteria.

Strategic Blueprinting: The coefficients of weightage conduce to the equitable allocation of resources and exertions within the ambit of strategic blueprinting. By focusing on criteria associated with higher weights, a more customized array of strategies and courses of action can be revealed, aligning with the overarching strategic goals. This acts as a catalyst for the development of strategic roadmaps aimed at enhancing the overall performance framework of the entity or system.

4. CASE STUDY

4.1. Computational Analyses

For the recommended method, a dataset comprising values of the Global Innovation Index (GII) criteria for the 19 countries in the G20 group for the year 2022 has been provided, along with an identity matrix ranging from 1. The corresponding decision matrix is presented in Table 2.

Table 2. Decision matrix

Countries	GII1	GII2	GII3	GII4	GII5	GII6	GII7
Argentina	42.6	30.5	44	24.9	31.2	19	24.2
Australia	77.2	61.7	58.8	50.2	48.6	32.2	37.8
Brazil	46.7	36.2	43.9	37.2	37.9	24.8	24.5
Canada	80.4	57.7	57	65.1	52.3	39.3	38.7
China	64.8	53.1	57.5	56	55.9	56.8	49.3
France	77	57.3	59	58	53.2	45.5	52.5
Germany	76.5	64.1	57.7	53.7	52.7	54.8	52.3
India	60.1	38.3	40.7	50.3	30.9	33.8	24.3
Indonesia	55.1	22.4	43.4	41.7	22.1	19	18.6
Italy	59	46.8	57.4	41.9	39.3	45.2	41.3
Japan	75.8	52.7	61.3	59	58.1	52.6	38.9
Korea	70.5	66.4	60.3	48	58	54.7	55.1
Mexico	48.2	33.6	44.2	36.3	25.2	24.3	24.7
Russia	48.7	47	44.3	37.4	35.4	26.6	25.3
Saudi Arabia	60.6	45.6	48	47	31	21	19.5
South Africa	51.9	26.9	40.7	40.4	27.6	24.7	19.5
Türkiye	46.8	38.9	49.2	41.6	32.5	24.7	41.5
United Kingdom	74.5	61.5	62.9	67.6	51.7	55.7	55.9
USA	80.9	59.9	58.7	80.8	64.5	60.8	48.4
MİN	42.6	22.4	40.7	24.9	22.1	19	18.6
MAK	80.9	66.4	62.9	80.8	64.5	60.8	55.9

Continuing with the proposed method, the normalized values of the decision matrix were computed using Equation 2. The measured normalized values are presented in Table 3.

Table 3. Normalized values

Countries	GII1	GII2	GII3	GII4	GII5	GII6	GII7
Direction	**Mak.**	**Mak.**	**Mak.**	**Mak.**	**Mak.**	**Mak.**	**Mak.**
Argentina	0	0.184091	0.148649	0	0.214623	0	0.150134
Australia	0.903394	0.893182	0.815315	0.452594	0.625	0.315789	0.514745
Brazil	0.10705	0.313636	0.144144	0.220036	0.372642	0.138756	0.158177
Canada	0.986945	0.802273	0.734234	0.719141	0.712264	0.485646	0.538874
China	0.579634	0.697727	0.756757	0.556351	0.79717	0.904306	0.823056
France	0.898172	0.793182	0.824324	0.592129	0.733491	0.633971	0.908847
Germany	0.885117	0.947727	0.765766	0.515206	0.721698	0.856459	0.903485
India	0.456919	0.361364	0	0.454383	0.207547	0.354067	0.152815
Indonesia	0.326371	0	0.121622	0.300537	0	0	0
Italy	0.428198	0.554545	0.752252	0.304114	0.40566	0.626794	0.608579
Japan	0.866841	0.688636	0.927928	0.610018	0.849057	0.803828	0.544236
Korea	0.72846	1	0.882883	0.413238	0.846698	0.854067	0.978552
Mexico	0.146214	0.254545	0.157658	0.203936	0.073113	0.126794	0.163539
Russia	0.159269	0.559091	0.162162	0.223614	0.313679	0.181818	0.179625
Saudi Arabia	0.469974	0.527273	0.328829	0.395349	0.209906	0.047847	0.024129
South Africa	0.24282	0.102273	0	0.277281	0.129717	0.136364	0.024129
Türkiye	0.109661	0.375	0.382883	0.298748	0.245283	0.136364	0.613941
United Kingdom	0.832898	0.888636	1	0.763864	0.698113	0.87799	1
USA	1	0.852273	0.810811	1	1	1	0.798928
MİN	0	0	0	0	0	0	0
MAK	1	1	1	1	1	1	1

In continuation of the CEBM method, cubic functions were formulated taking into account the relationships between the criteria as indicated by Equations 3, 4, 5, and 6. Correspondingly, the cubic functions established based on the interrelations among the criteria are presented in Table 4.

Table 4. Cubic functions generated based on the relationship between criteria

x	y	Cubic Equations	x	y	Cubic Equations
GII1→	GII2	$y=-0.822+2.46x+0x^2-0.979x^3$	GII4→	GII1	$y=-0.065+0.600x+3.190x^2-2.779x^3$
	GII3	$y=0.229+1.877x+0x^2-0.738x^3$		GII1	$y=0.146+0.285x+2.794x^2-2.410x^3$
	GII4	y=0.424+ 0x-0.588x2+1.05x3		GII3	$y=0.110 -0.570x+5.338x^2-4.091x^3$
	GII5	y=-1.357+3.687x+0x2-1.674x3		GII5	$y=0.176-0.598x+4.146x^2-2.790x^3$
	GII6	$y=-0.407+1.512x+0x^2-0.176x^3$		GII6	$y=-0.0390+0.445x+2.418x^2-1.877x^3$
	GII7	$y=-1.490+3.982x +0x^2 -1.826x^3$		GII7	$y=0.113-0.372x+4.295x^2-3.263x^3$
GII2→	GII1	$y=0.386-2.905x+8.951x^2-5.800x^3$	GII5→	GII1	$y=0.314-1.532x+5.643x^2-3.543x^3$
	GII3	$y=0.136-1.380x+5.651x^2-3.568x^3$		GII2	$y=0.043+1.339x+0.227x^2-0.791x^3$
	GII4	$y=0.342-2.071x+6.406x^2-4.177x^3$		GII3	$y=0.101-0.375x+4.242x^2-3.190x^3$
	GII5	$y=0.084-0.429x+3.153x^2-2.009x^3$		GII4	$y=0.275-0.239x+0.893x^2-0.044x^3$
	GII6	$y=0.054-0.527x+3.244x^2-1.969x^3$		GII6	$y=0.046+0.190x+1.499x^2-0.718x^3$
	GII7	$y=-0.005+0.512x+0.402x^2+0.052x^3$		GII7	$y=0.033+0.240x+2.627x^2-2.126x^3$
GII3→	GII1	$y=-0.343-2.069x+6.412x^2-3.860x^3$	GII6→	GII1	$y=0.117+1.390x-0.414x^2--0.279x^3$
	GII2	$y=0.207+0.270x+1.461x^2-1.075x^3$		GII2	$y=0.143+1.916x-2.047x^2+0.866x^3$
	GII4	$y=0.326-1.049x+3.064x^2-1.680x^3$		GII3	$y=0.164-0.158x+3.630x^2-2.874x^3$
	GII5	$y=0.195-0.656x+3.281x^2-2.028x^3$		GII4	$y=0.130+1.841x-3.700x^2+2.582x^3$
	GII6	$y=0.250-1.941x+5.602x^2-3.055x^3$		GII5	$y=0.089+1.370x-1.372x^2+0.850x^3$
	GII7	$y= 0.069+0.259x+1.499x^2-0.949x^3$		GII7	$y=0.085+0.570x+1.417x^2-1.214x^3$
GII7→	GII1	$y=0.304-1.020x+4.578x^2-3.098x^3$			
	GII2	$y=0.137+1.858x-2.465x^2+1.418x^3$			
	GII3	$y=0.0800+0.450x+1.953x^2-1.635x^3$			
	GII4	$y=0.326-1.144x+4.083x^2-2.700x^3$			
	GII5	$y=0.091+0.915x+0.288x^2-0.517x^3$			
	GII6	$y=0.071+ 0.202x+1.661x^2-1.060x^3$			

x=Independent Veariable. y=Dependent Variable

In the third phase of the approach, cubic influence factors among the criteria were computed utilizing equations 6, 7, 8, and 9. The computation process for the impact values of the GII1 criterion on the remaining criteria is elucidated in the following sections. The determination of impact values for the remaining components of GII can be found in Appendix A, provided for reference.

f(GII1)=GII2

$$f(x) = y = 0{,}357 - 1{,}528x + 5{,}574xx - 3{,}599xxx$$

$$f'(x) = \frac{-10797x^2}{1000} + \frac{2787x}{250} - 1{,}528$$

$$\int_0^1 \frac{-10797x^2}{1000} + \frac{2787x}{250} - 1{,}528dx = \frac{447}{1000} = 0{,}447$$

• ***f(GII1)=GII3***

$$f(x) = y = 0.289 - 2.163x + 7.606xx - 4.984xxx$$

$$f'(x) = \frac{-1869x^2}{125} + \frac{3803x}{250} - 2{,}163$$

$$\int_0^1 \frac{-1869x^2}{125} + \frac{3803x}{250} - 2{,}163dx = 0{,}459$$

• ***f(GII1)=GII4***

$$f(x) = y = 0{,}0227 + 1{,}952x - 3{,}631xx + 2{,}472xxx$$

$$f'(x) = \frac{927x^2}{125} - \frac{3631x}{500} + 1{,}952$$

$$\int_0^1 \frac{927x^2}{125} - \frac{3631x}{500} + 1{,}952dx = \frac{9432466197}{20000000000} = 0{,}793$$

• ***f(GII1)=GII5***

$$f(x) = y = 0{,}3251 - 1{,}7902x + 5{,}489xx - 3{,}238xxx$$

$$f'(x) = \frac{-4857x^2}{500} + \frac{5489x}{500} - 1{,}7902$$

$$\int_0^1 \frac{-4857x^2}{500} + \frac{5489x}{500} - 1{,}7902dx = 0{,}461$$

• ***f(GII1)=GII6***

$$f(x) = y = 0{,}095 - 0{,}656x + 4{,}499xx - 3{,}288xxx$$

$$f'(x) = \frac{-1233x^2}{125} + \frac{4499x}{500} - 0{,}656$$

$$\int_0^1 \frac{-1233x^2}{125} + \frac{4499x}{500} - 0{,}656dx = 0{,}555$$

• ***f(GII1)=GII7***

$$f(x) = y0{,}386 - 2{,}905x + 8{,}951xx - 5{,}800xxx$$

$$f'(x) = \frac{-87x^2}{5} + \frac{8951x}{500} - 2{,}905$$

$$\int_0^1 \frac{-87x^2}{5} + \frac{8951x}{500} - 2{,}905dx = 0{,}246$$

In the fourth step of the process, the cumulative cubic impact values for each criterion were computed using formulas 10, 11, 12, and 13. These calculated values have been displayed in Table 5.

Table 5. Sum of cubic impact values of GII components on each other

Independent Component	Dependent Components	Effect	Absolute Value	Independent Component	Dependent Components	Effect	Absolute Value
GII1→	**GII2**	0.447	0.447	GII4→	**GII1**	0.999	0.999
	GII3	0.459	0.459		**GII2**	0.669	0.669
	GII4	0.793	0.793		**GII3**	0.667	0.667
	GII5	0.461	0.461		**GII5**	0.758	0.758
	GII6	0.555	0.555		**GII6**	0.986	0.986
	GII7	0.246	0.246		**GII7**	0.66	0.66
	Total	2.961	2.961		**Total**	4.739	4.739
GII2→	**GII1**	0.246	0.246	GII5→	**GII1**	0.568	0.568
	GII3	0.703	0.703		**GII2**	0.775	0.775
	GII4	0.158	0.158		**GII3**	0.677	0.677
	GII5	0.715	0.715		**GII4**	0.61	0.61
	GII6	0.748	0.748		**GII6**	0.971	0.971
	GII7	0.966	0.966		**GII7**	0.741	0.741
	Total	3.536	3.536		**Total**	4.342	4.342
GII3→	**GII1**	0.483	0.483	GII6→	**GII1**	0.697	0.697
	GII2	0.656	0.656		**GII2**	0.735	0.735
	GII4	0.335	0.335		**GII3**	0.598	0.598
	GII5	0.597	0.597		**GII4**	0.723	0.723
	GII6	0.606	0.606		**GII5**	0.848	0.848
	GII7	0.809	0.809		**GII7**	0.773	0.773
	Total	3.486	3.486		**Total**	4.374	4.374

Independent Component	Dependent Components	Effect Value	Absolute Value
GII7→	**GII1**	0.460	0.46
	GII2	0.811	0.811
	GII3	0.768	0.768
	GII4	0.239	0.239
	GII5	0.686	0.686
	GII6	0.803	0.803
	Total	3.767	**3.767**

Moreover, in Equation 14, the weight coefficients, which represent the levels of significance for each criterion, are computed. These coefficients quantify the relative importance of the criteria within the context of the analysis. The resulting values have been documented in Table 6.

Table 6. Weighting coefficients of the components

GII Components	Total Effects	w	Ranking
GII1	2.961	0.1088403	7
GII2	3.536	0.1299761	5
GII3	3.486	0.1281382	6
GII4	4.739	0.1741959	1
GII5	4.342	0.159603	3
GII6	4.374	0.1607793	2
GII7	3.767	0.1384672	4
Toplam	27.205	------	-----

Upon thorough examination of Table 4, the significance assigned to the diverse constituents of the GII (Cubic Impact) has been arranged as follows: GII4 holds the highest weight coefficient, succeeded by GII4, GII6, GII5, GII7, GII2, GII3, and finally GII1. This sequence elucidates the varying levels of importance attributed to each constituent within the GII framework.

4.2. Computational Analyses

Within the scope of this research, an examination of the CEBM method was conducted to assess its sensitivity in terms of methodology. Sensitivity analysis, in the context of MCDA, involves a process where various criteria weighting methods are applied to the same dataset, facilitating a comparison of the resulting values and rankings. To ensure the sensitivity of the weight coefficient calculation method, the weight ranking of the criteria identified with the method to be subjected to sensitivity analysis is expected to be different from the weight coefficient rankings identified with other methods (Gigovič, 2016).

In accordance with this approach, for the purpose of sensitivity analysis, the weighting coefficients associated with the components of the GII were calculated and organized using well-established objective weighting techniques prevalent in scholarly literature. Noteworthy examples of these techniques encompass ENTROPY, CRITIC, SD (Standard Deviation), SVP

(Statistical Variance Procedure), MEREC, and LOPCOW. The corresponding numerical outcomes have been meticulously documented in Table 7.

Table 7. Values for other methods of calculating objective weighting coefficients

GII	ENTROPY		CRITIC		SD	
	Value	Ranking	Value	Ranking	Value	Ranking
GII1	0.0932089	6	0.197686	1	0.1082728	6
GII2	0.1785586	4	0.1294088	5	0.14443	4
GII3	0.0492509	7	0.1237116	6	0.0794983	7
GII4	0.1471809	5	0.1578584	3	0.1362273	5
GII5	0.2030259	3	0.0782892	7	0.1556881	3
GII6	0.3287748	1	0.1481991	4	0.1941804	1
GII7	0.2872703	2	0.1648469	2	0.1817031	2
GII	**SVP**		**LOPCOW**		**MEREC**	
	Value	**Ranking**	**Value**	**Ranking**	**Value**	**Ranking**
GII1	0.1771006	4	0.1430929	4	0.0887702	6
GII2	0.184283	2	0.1609176	1	0.148061	4
GII3	0.0639185	7	0.1360646	5	0.0488366	7
GII4	0.1760155	5	0.1608688	2	0.176495	3
GII5	0.1745698	6	0.1438358	3	0.198777	2
GII6	0.2241127	1	0.1249748	7	0.2161803	1
GII7	0.1804056	3	0.1302454	6	0.1228798	5

When Tables 6 and 7 are compared simultaneously, it becomes evident that the prioritization of criteria weighting coefficients for the Global Innovation Index (GII) varies when determined through the CEBM in comparison to other methods. This shows that the CEBM method is a sensitive technique.

4.3. Computational Analyses

In the comparative analysis, the relationships and positions of the proposed method with other objective weight coefficient calculation methods are evaluated. In this regard, it is expected that the proposed method is credible and reliable, and does not differ much from other methods, and has a positive and significant relationship with different weight coefficient methods (Keshavarz-Ghorabaee et al., 2021). Based on the data shown in Table 7, the positions of the methods are shown in Figures 1 and 2.

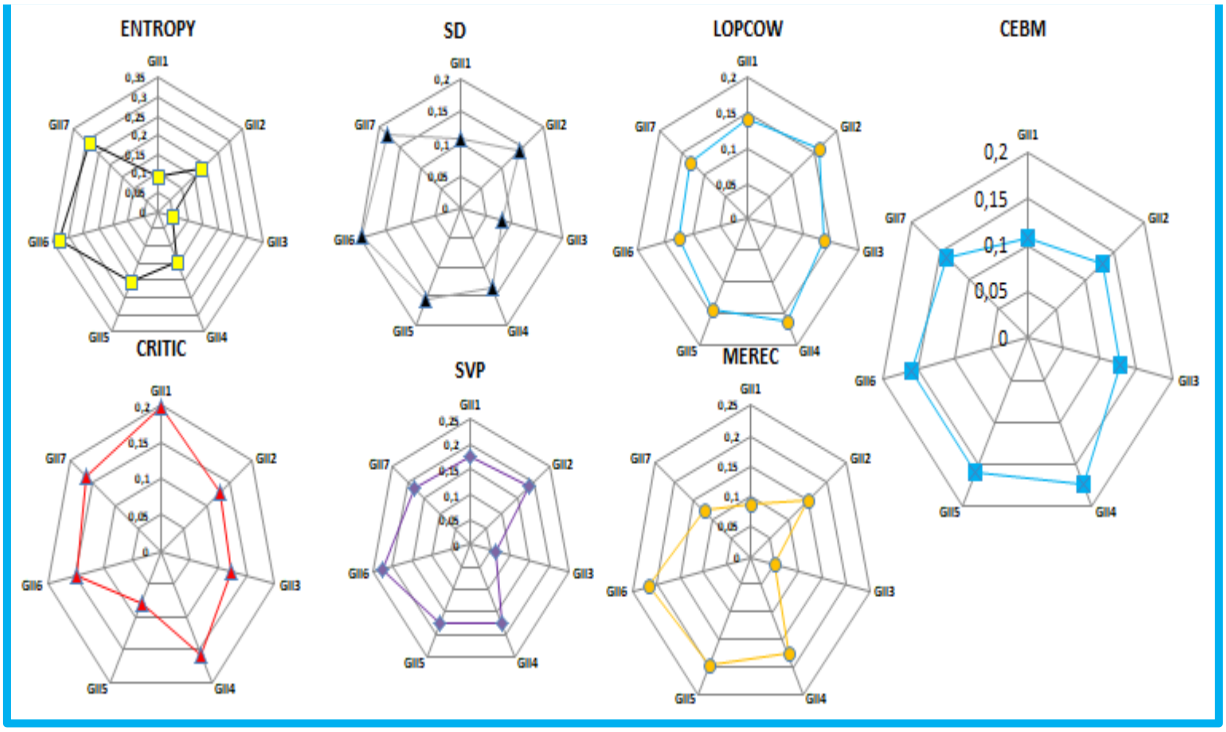

Figure 1. Positions of the methods-1

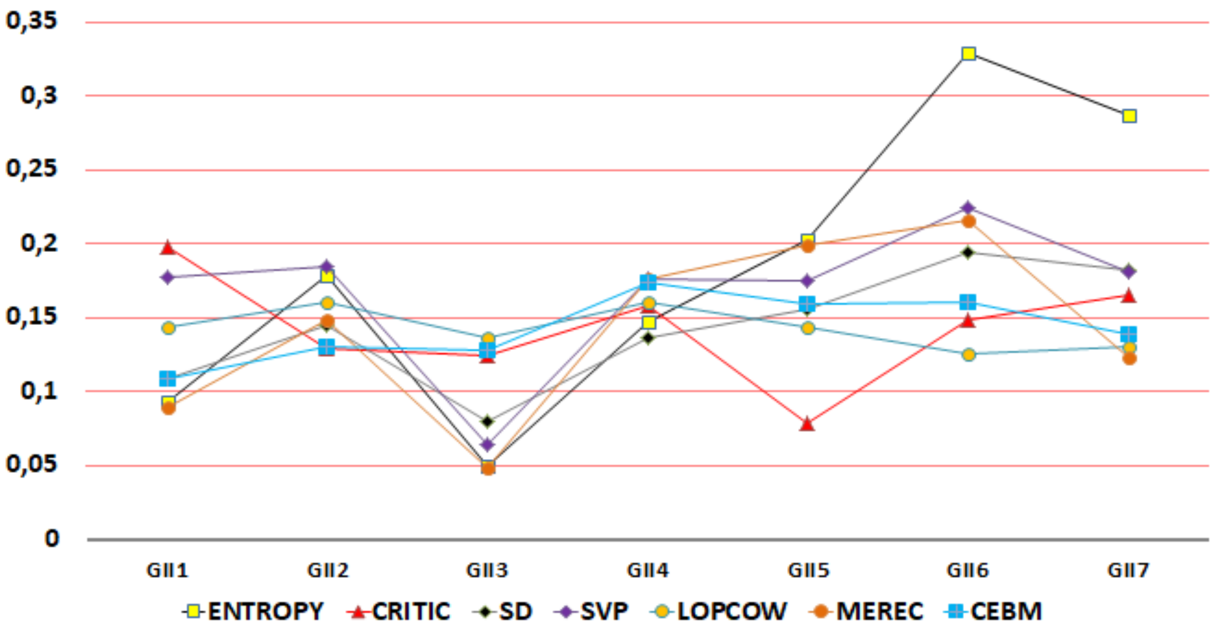

Figure 2. Positions of the methods-2

According to Figure 1, the proportional similarity of the point locations of the CEBM method to the MEREC method is greater than that of the other methods. In addition, in Figure 2, the differences between the CEBM method and MEREC points are at a lower level than the differences between the CEBM method and the points of other methods. In light of all this data, it can be evaluated that the relationships between the CEBM method and the MEREC method are positive, significant, and high. In this regard, the correlation values of the CEBM method with other methods are shown in Table 8.

Table 8. Correlation values of the CEBM method with other Methods

Methods	ENTROPY	CRITIC	SD	SVP	LOPCOW	MEREC
CEBM	0.470	-0.392	0.519*	0.343	0.091	0.791**

p<.01. p*<.05**

According to Table 8, the CEBM method has a significant, positive, moderate relationship with the SD method, and a significant, positive, and high relationship with the MEREC method. In this regard, the fact that the CEBM method has significant positive relationships with the SD and MEREC methods suggests that the method is credible and reliable.

4.4. Simulation Analysis

To ensure the simulation analysis, different scenarios are created by assigning different quantities to decision matrices. For the stability of results determined by proposed method, the proposed method is expected to differ from other methods as the number of scenarios increases. In the second case, the average of the variance values of the proposed method according to the scenarios must be greater than one or several of the other objective weighting methods. This shows that the proposed method is relatively effective in distinguishing the weights of the criteria. Finally, in the fourth case, the homogeneity of the variances of the criterion weights according to the methods within the scenarios must be formed (Keshavarz-Ghorabaee vd, 2021).

In the simulation analysis, the correlation values of the CEBM method with other methods were calculated according to the 10 scenarios created first and are shown in Table 9.

Table 9. Correlation values of the CEBM method with other methods within scenarios.

Group	Scenarios	ENTROPY	CRITIC	SD	SVP	LOPCOW	MEREC
First group	1. Scenario	0.493	-0.410	0.524*	0.312	0.053	0.810**
	2. Scenario	0.475	-0.470	0.600*	0.345	0.065	0.800**
	3. Scenario	0.512*	-0.455	0.640*	0.325	0.078	0.843**
Second group	Scenarios	ENTROPY	CRITIC	SD	SVP	LOPCOW	MEREC
	4. Scenario	0.535	-0.443	0.480*	0.382	0.065	0.754**
	5. Scenario	0.464	-0.385	0.300	0.355	0.064	0.766**
	6. Scenario	0.445	-0.475	0.250	0.205	0.052	0.615*
	7. Scenario	0.523*	-0.510*	0.240	0.295	0.025	0.623*
	8. Scenario	0.495	-0.630*	0.215	0.343	0.035	0.700*
	9. Scenario	0.277	-0.420	0.270	0.232	0.052	0.599*
	10. Scenario	0.435	-0.600*	0.420	0.315	0.065	0.625*
	Ortalama	0.465	-0.481	0.499	0.326	0.118	0.797**

p<.01. p**<.05**

According to Table 9, it is evaluated that the criterion weights differ from each other according to the methods as the number of scenarios increases. In addition, the positive and significant relationships of the CEBM method with the MEREC method in all scenarios are noteworthy. The data shown in Table 7 were divided into two groups, and the comparison of the correlation values between the created groups is shown in Figure 3.

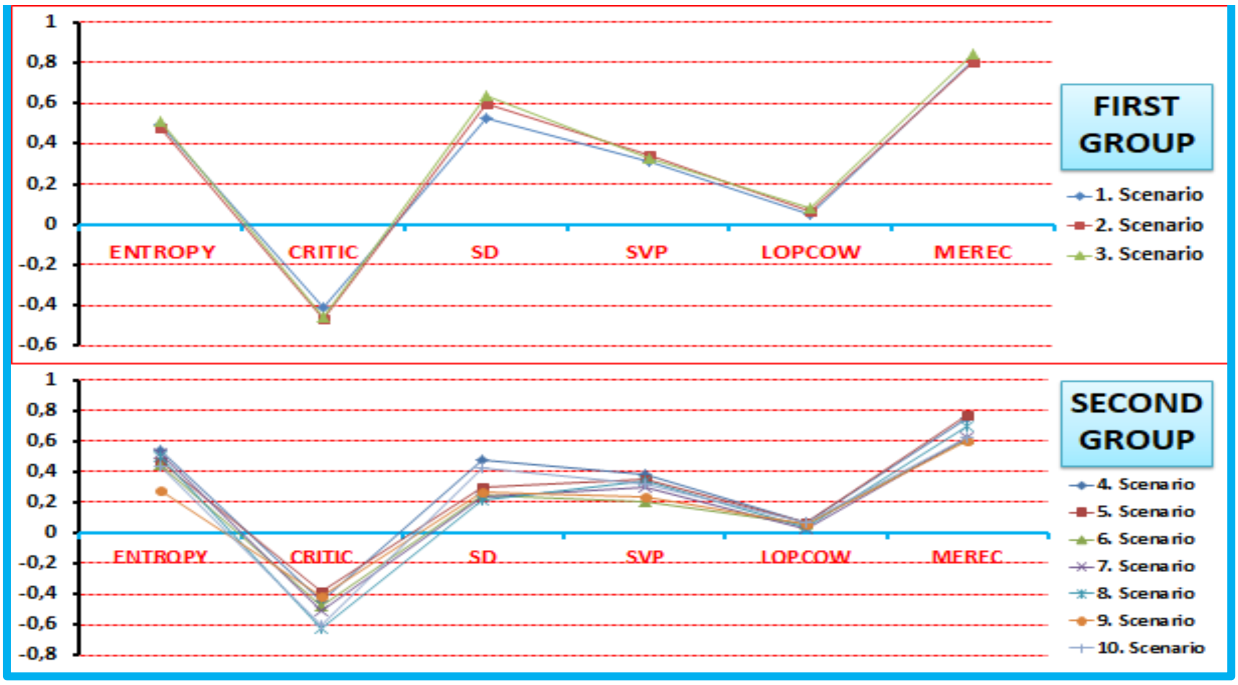

Figure 3. Correlation status of the CEBM method with other methods within scenarios

According to Figure 3, the correlation status of the CEBM method with other methods generally differs as the number of scenarios increases. This difference is evaluated as having a decreasing effect on the correlation value for the ENTROPY method in the 9th scenario, an increasing effect on the negative correlation value for the CRITIC method in the 7th, 8th, 9th, and 10th scenarios, a decreasing effect on the correlation value for the SD and MEREC methods after the 3rd scenario, a decreasing effect on the correlation value for the SVP method in the 9th and 10th scenarios, and a general decreasing effect on the correlation value for the LOPCOW method in all scenarios. The discriminant image of the correlation values of the CEBM method with other methods in terms of scenarios is presented in Figure 5.

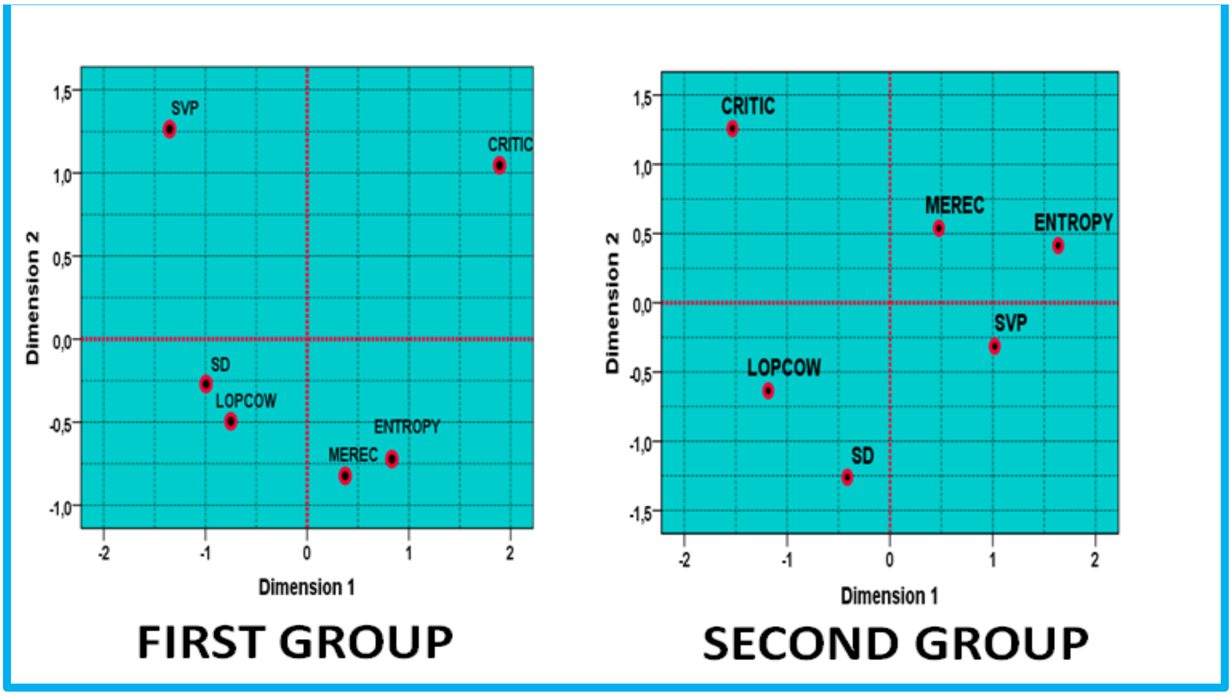

Figure 4. Discriminant image of the correlation status of the CEBM method with other methods in terms of scenarios

According to Figure 4, in the first group, methods are generally close to each other in the first three scenarios. However, it has been determined that the methods are generally distant from each other in the next 7 scenarios. Accordingly, it has been determined that the characteristic qualities of the methods become more pronounced as the scenarios increase and that the methods therefore become more distant from each other. In the simulation analysis, the variance values of the methods were calculated within the scenarios, and the calculated values are presented in Table 10.

Table 10. Variance values of methods by scenarios

Scenario	CEBM	ENTROPY	CRITIC	SD	SVP	LOPCOW	MEREC
1. Scenario	0.0035238	0.002962	0.00141	0.002594	0.002431	0.0021961	0.0036377
2. Scenario	0.0031365	0.0038422	0.003349	0.002448	0.002137	0.0026993	0.0025125
3. Scenario	0.002196	0.0031448	0.002247	0.003869	0.002534	0.002508	0.0022544
4. Scenario	0.005001	0.0021184	0.001582	0.000946	0.002893	0.0032147	0.0049132
5. Scenario	0.001898	0.0031889	0.001468	0.001628	0.002533	0.0028489	0.0033628
6. Scenario	0.002261	0.0031394	0.002833	0.0012	0.002659	0.0022608	0.0022812
7. Scenario	0.003517	0.0026188	0.003401	0.00391	0.002161	0.0032112	0.0032689
8. Scenario	0.0029902	0.0029758	0.001869	0.002692	0.002775	0.0038766	0.0027456
9. Scenario	0.0032298	0.0038046	0.002468	0.00141	0.000252	0.0033576	0.0027567
10. Scenario	0.0022397	0.0022693	0.001	0.001262	0.002186	0.0037955	0.0029275
Mean	0.0029993	0.00300642	0.002163	0.002196	0.002256	0.00299687	0.00306605

According to Table 10, the average variance value of the CEBM method is lower than the average variance values of the MEREC and ENTROPY methods, but higher than the average variance values of the CRITIC, SD, and SVP methods. Again, according to Table 8, the average variance values of the CEBM and LOPCOW methods are found to be close to each other. Therefore, it can be evaluated that the CEBM method is relatively effective in distinguishing the weights of criteria, as the average variance value of the CEBM method is higher than the average variance values of the CRITIC, SD, and SVP methods.

In the continuation of the simulation analysis, the homogeneity of the variances of the criterion weights of the CEBM method was analyzed by ADM (ANOM for variances with Levene) analysis within the scenarios. This analysis is an analysis that helps us to obtain a graphical representation to verify the homogeneity of the variances. The graphical representation has three variables: the general average ADM is the center line, the upper decision limits (UDL) and the lower decision limits (LDL). If the standard deviation of a group (cluster) falls outside the decision limits, that standard deviation is significantly different from the general average ADM and there is heterogeneity in the variances. In other words, if the standard deviations of all clusters are between LDL and UDL, the homogeneity of the variances is verified (Keshavarz-Ghorabaee et al., 2021). The visual for the ADM analysis is shown in Figure 5.

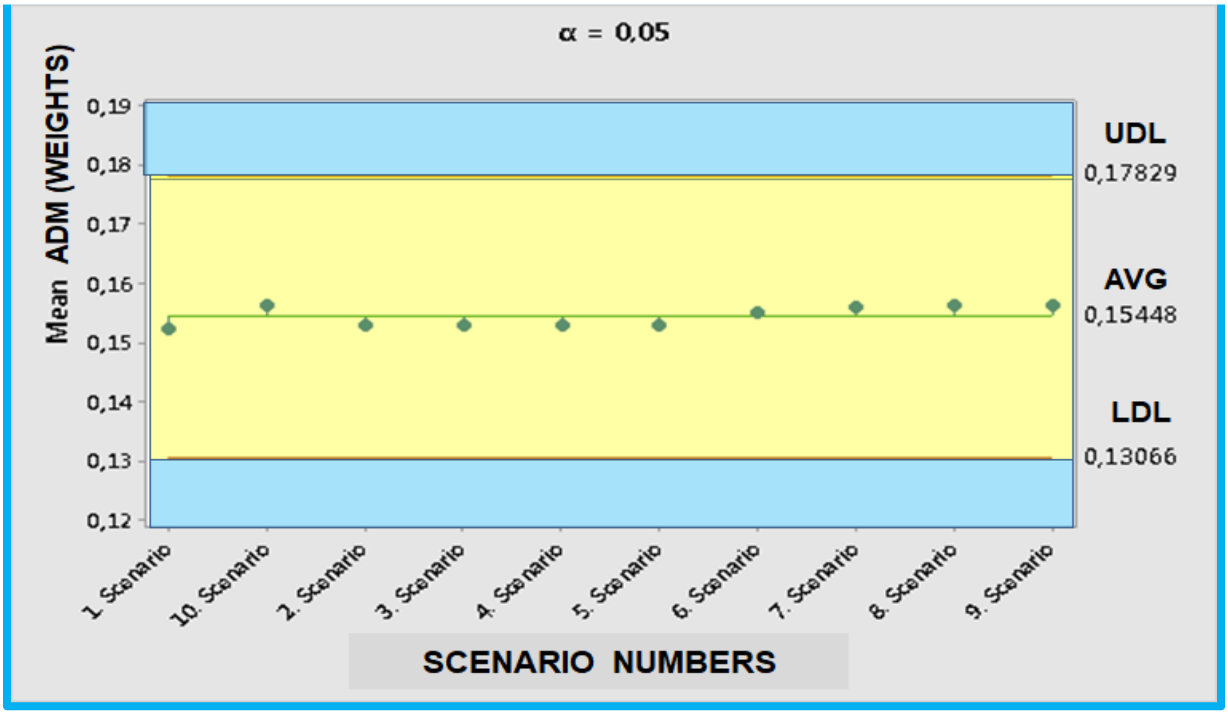

Figure 5. ADM Visual

As shown in Figure 5, the ADM values generated for each scenario fall below the UDL values and above the LDL values. Therefore, the variances of the weights identified for each scenario are homogeneous. This condition was also measured by the Levene Test. The basic values for the Levene Test are presented in Table 11.

Table 11. Levene Statistic

Levene Statistic	df1	df2	Sig.
0.522	2	10	0.174

p<.05**

According to Table 11, the p-value (p=0.174) is greater than 0.05, so the variances of the criterion weights across scenarios are homogeneous. In general, the simulation analysis results indicate that the CEBM method is robust and stability.

5. RESULTS AND DISCUSSION

Multi-criteria decision making is a prevalent approach utilized to address intricate decision conundrums. This method strives to select from various options by taking into account a set of different standards. Nevertheless, the significance of each criterion might diverge, underscoring the need to assign weights to these criteria. Assigning these weights serves to cultivate an impartial and unbiased process in the decision-making framework, thereby elucidating the interconnections and priorities among the distinct criteria. This, in turn, facilitates the attainment of more coherent and dependable outcomes throughout the decision-making procedure. Consequently, numerous scholars have devised novel techniques to compute the weight factors for these criteria. Each approach has augmented the spectrum of knowledge in Multiple Criteria Decision Making (MCDM) by employing diverse methodologies.

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Furthermore, as novel methodologies for determining the weight coefficients of criteria persistently surface, there is an escalating trend toward specialization in the computation of these weights. As a result, in this research endeavor, a fresh methodology grounded in cubic functions (CEBM) is introduced for the computation of weight coefficients attributed to the various criteria.The The study's dataset encompassed data from the Global Innovation Index (GII) for the year 2023, focusing on 19 countries belonging to the G20 coalition. Initially, CEBM was employed to compute the weight coefficients of the GII's constituent elements.

In the study, the weight values of the GII criteria were calculated using other objective criterion weighting methods (ENTROPY, CRITIC, SVP, SD, LOPCOW and MEREC) to measure the sensitivity of the proposed method, and the GII criterion weight ranking identified within the CEBM method was compared with the other objective criterion weighting methods. According to the findings, the weight ranking of the GII criteria determined by the CEBM method completely differed from the weight coefficient rankings of the GII criteria determined by the other objective criterion weighting methods. Based on this result, it was concluded that the proposed method is sensitive.

In the study, the second approach was the comparative analysis of the CEBM method. Accordingly, the similarity of the CEBM method with other objective weight methods was analyzed. According to the results, the CEBM method was found to have a positive, significant and high relationship with the MEREC method, and a positive, significant and medium-level relationship with the SD method. In general, it was observed that the CEBM method does not have a very high similarity with other criterion weighting methods in general. Based on these results, it was concluded that the CEBM method is credible and reliable.

In the research, within the scope of a simulation analysis, ten different GII decision matrices were created using the CEBM method and other objective weighting coefficient methods. These matrices were categorized into two groups: the first group comprising 3 scenarios and the second group comprising 7 scenarios. In this context, the correlation values between the CEBM method in the first and second groups and other objective weighting methods were compared. The findings indicated that as the number of scenarios increased, the correlation values between the CEBM method and other methods generally decreased, suggesting that the distinctive characteristics of the CEBM method became more pronounced. Secondly, in the simulation analysis, the variance values of the methods were calculated within the scenarios. According to the results, the CEBM method's average variance value was higher than the average variance values of the CRITIC, SD, and SVP methods. This implies that the CEBM method is relatively effective in distinguishing the weights of criteria. Continuing with the simulation analysis, the homogeneity of variance for the CEBM method's criterion weights within the scenarios was assessed using the ADM analysis. The ADM values created according to the scenarios were observed to be below UDL values and above LDL values. Furthermore, the homogeneity of variances for the CEBM method was measured using the Levene test. Since the significance value in this test was greater than 0.05, it was concluded that the variances of the methods were homogeneous. Therefore, based on the simulation analysis data, it was determined that the CEBM method is stable and robust.

Just as the CEBM method has its advantages, it also has some disadvantages and limitations. One notable drawback or limitation is its intricate computational process for determining criteria weight coefficients, especially when the number of criteria expand The complexity arises from the multitude of interaction values between criteria. Another drawback or limitation is its dependency on statistical software tools like SPSS to identify cubic relationships between criteria. Should one lack access to SPSS, the weight coefficient calculations according to this method become more convoluted and time-consuming. Moreover, a third drawback or limitation arises when a clear cause-and-effect relationship is absent among the criteria. This situation can restrict opportunities for improving the criteria using this approach.

Considering all these findings, it can be concluded that the CEBM method exhibits a high level of sensitivity, credible, reliable, and stability. In summary, this study aims to demonstrate the feasibility of quantifying criterion weights using the CEBM within the realm of MCDM literature. The proposed approach is expected to provide a valuable tool for objectively assessing the effectiveness of available decision options. The outcomes of this research hold significant implications for scholars and decision-makers operating within this relevant field. Therefore, it is anticipated that these research findings will stimulate an increased emphasis on the integration of cubic functions in mathematical modeling processes across academic circles, corporate environments, and other institutional settings. Furthermore, it can be inferred that the CEBM method stands out as an effective resource for decision-makers involved in complex tasks of choice and judgment, particularly related to the performance evaluation of various decision alternatives.

ETHICAL DECLARATION

In the writing process of the study titled “A novel approach to measuring criterion weights in multiple criteria decision making: cubic effect-based measurement (CEBM)”, there were followed the scientific, ethical and the citation rules; was not made any falsification on the collected data and this study was not sent to any other academic media for evaluation.

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Appendix A

F(GII2)

F(GII2)=GII1

$$f(x) = y = 0{,}386 - 2{,}905x + 8{,}951x^2 - 5{,}800x^3$$

$$f'(x) = \frac{-87x^2}{5} + \frac{8951x}{500} - 2{,}905$$

$$\int_0^1 \frac{-87x^2}{5} + \frac{8951x}{500} - 2{,}905dx = 0{,}246$$

F(GII2)=GII3

$$f(x) = y = 0{,}136 - 1{,}380x + 5{,}651x^2 - 3{,}568x^3$$

$$f'(x) = \frac{-1338x^2}{125} + \frac{5651x}{500} - 1{,}38$$

$$\int_0^1 \frac{-1338x^2}{125} + \frac{5651x}{500} - 1{,}38dx = 0{,}703$$

F(GII2)=GII4

$$f(x) = y = 0{,}342 - 2{,}071x + 6{,}406x^2 - 4{,}177x^3$$

$$f'(x) = \frac{-12531x^2}{1000} + \frac{3203x}{250} - 2{,}071$$

$$\int_0^1 \frac{-12531x^2}{1000} + \frac{3203x}{250} - 2{,}071dx = 0{,}158$$

F(GII2)=GII5

$$f(x) = y = 0{,}084 - 0{,}429x + 3{,}153x^2 - 2{,}009x^3$$

$$f'(x) = \frac{-6027x^2}{1000} + \frac{3153x}{500} - 0{,}429$$

$$\int_0^1 \frac{-6027x^2}{1000} + \frac{3153x}{500} - 0{,}429dx = 0{,}715$$

F(GII2)=GII6

$$f(x) = y = 0{,}054 - 0{,}527x + 3{,}244x^2 - 1{,}969x^3$$

$$f'(x) = \frac{-5907x^2}{1000} + \frac{811x}{125} - 0{,}527$$

$$\int_0^1 \frac{-5907x^2}{1000} + \frac{811x}{125} - 0{,}527dx = 0{,}748$$

F(GII2)=GII7-$f(x) = y = 0{,}005 + 0{,}512x + 0{,}402x^2 + 0{,}052x^3$

$$f'(x) = \frac{39x^2}{250} + \frac{201x}{250} + 0{,}512$$

$$\int_0^1 \frac{39x^2}{250} + \frac{201x}{250} + 0{,}512dx = 0{,}966$$

F(GII3)

F(GII3)=GII1

$$f(x) = y = 0{,}343 - 2{,}069x + 6{,}412x^2 - 3{,}860x^3$$

$$f'(x) = \frac{-579x^2}{50} + \frac{1603x}{125} - 2{,}069$$

$$\int_0^1 \frac{-579x^2}{50} + \frac{1603x}{125} - 2{,}069dx = 0{,}483$$

F(GII3)=GII2

$$f(x) = y = 0{,}207 + 0{,}270x + 1{,}461x^2 - 1{,}075x^3$$

$$f'(x) = \frac{-129x^2}{40} + \frac{1461x}{500} + 0{,}27$$

$$\int_0^1 \frac{-129x^2}{40} + \frac{1461x}{500} + 0{,}27dx = 0{,}656$$

F(GII3)=GII4

$$f(x) = y = 0{,}326 - 1{,}049x + 3{,}064x^2 - 1{,}680x^3$$

$$f'(x) = \frac{-126x^2}{25} + \frac{766x}{125} - 1{,}049$$

$$\int_0^1 \frac{-126x^2}{25} + \frac{766x}{125} - 1{,}049dx = 0{,}335$$

F(GII3)=GII5

$$f(x) = y = 0{,}195 - 0{,}656x + 3{,}281x^2 - 2{,}028x^3$$

$$f'(x) = \frac{-1521x^2}{250} + \frac{3281x}{500} - 0,656$$

$$\int_0^1 \frac{-1521x^2}{250} + \frac{3281x}{500} - 0,656dx = 0,597$$

F(GII3)=GII6

$$f(x) = y = 0,250 - 1,941x + 5,602x^2 - 3,055x^3$$

$$f'(x) = \frac{-1833x^2}{200} + \frac{2801x}{250} - 1,941$$

$$\int_0^1 \frac{-1833x^2}{200} + \frac{2801x}{250} - 1,941dx = 0,606$$

F(GII3)=GII7

$$f(x) = y = 0,069 + 0,259x + 1,499x^2 - 0,949x^3$$

$$f'(x) = \frac{-2847x^2}{1000} + \frac{1499x}{500} + 0,259$$

$$\int_0^1 \frac{-2847x^2}{1000} + \frac{1499x}{500} + 0,259dx = 0,809$$

F(GII4)

F(GII4)=GII1

$$f(x) = y = -0,065 + 0,600x + 3,190x^2 - 2,779x^3$$

$$f'(x) = \frac{-8337x^2}{1000} + \frac{319x}{50} + 0,6$$

$$\int_0^1 \frac{-8337x^2}{1000} + \frac{319x}{50} + 0,6dx = 1$$

F(GII4)=GII2

$$f(x) = y = 0,146 + 0,285x + 2,794x^2 - 2,410x^3$$

$$f'(x) = \frac{-723x^2}{100} + \frac{1397x}{250} + 0,285$$

$$\int_0^1 \frac{-723x^2}{100} + \frac{1397x}{250} + 0,285dx = 0,669$$

F(GII4)=GII3

$$f(x) = y = 0,110 - 0,570x + 5,338x^2 - 4,091x^3$$

$$f'(x) = \frac{-12273x^2}{1000} + \frac{2669x}{250} - 0,57$$

$$\int_0^1 \frac{-12273x^2}{1000} + \frac{2669x}{250} - 0,57dx = 0,667$$

F(GII4)=GII5

$$f(x) = y = 0,176 - 0,598x + 4,146x^2 - 2,790x^3$$

$$f'(x) = \frac{-837x^2}{100} + \frac{2073x}{250} - 0,598$$

$$\int_0^1 \frac{-837x^2}{100} + \frac{2073x}{250} - 0,598dx = 0,758$$

F(GII4)=GII6

$$f(x) = y = -0,0390 + 0,445x + 2,418x^2 - 1,877x^3$$

$$f'(x) = \frac{-5631x^2}{1000} + \frac{1209x}{250} + 0,445$$

$$\int_0^1 \frac{-5631x^2}{1000} + \frac{1209x}{250} + 0,445dx = 0,986$$

F(GII4)=GII7

$$f(x) = y = 0,113 - 0,372x + 4,295x^2 - 3,263x^3$$

$$f'(x) = \frac{-9789x^2}{1000} + \frac{859x}{100} - 0,372$$

$$\int_0^1 \frac{-9789x^2}{1000} + \frac{859x}{100} - 0,372dx = 0,660$$

F(GII5)

$$f(x) = y = 0,314 - 1,532x + 5,643x^2 - 3,543x^3$$

$$f'(x) = \frac{-10629x^2}{1000} + \frac{5643x}{500} - 1,532$$

$$\int_0^1 \frac{-10629x^2}{1000} + \frac{5643x}{500} - 1,532dx = 0,568$$

F(GII5)=GII2

$f(x) = y = 0{,}043 + 1{,}339x + 0{,}227x^2 - 0{,}791x^3$

$f'(x) = \frac{-2373x^2}{1000} + \frac{227x}{500} + 1{,}339$

$\int_0^1 \frac{-2373x^2}{1000} + \frac{227x}{500} + 1{,}339dx = 0{,}775$

F(GII5)=GII3

$f(x) = y = 0{,}101 - 0{,}375x + 4{,}242x^2 - 3{,}190x^3$

$f'(x) = \frac{-957x^2}{100} + \frac{2121x}{250} - 0{,}375$

$\int_0^1 \frac{-957x^2}{100} + \frac{2121x}{250} - 0{,}375dx = 0{,}677$

F(GII5)=GII4

$(x) = y = 0{,}275 - 0{,}239x + 0{,}893x^2 - 0{,}044x^3$

$f'(x) = \frac{-33x^2}{250} + \frac{893x}{500} - 0{,}239$

$\int_0^1 \frac{-33x^2}{250} + \frac{893x}{500} - 0{,}239dx = 0{,}610$

F(GII5)=GII6

$(x) = y = 0{,}046 + 0{,}190x + 1{,}499x^2 - 0{,}718x^3$

$f'(x) = \frac{-1077x^2}{500} + \frac{1499x}{500} + 0{,}19$

$\int_0^1 \frac{-1077x^2}{500} + \frac{1499x}{500} + 0{,}19dx = 0{,}971$

F(GII5)=GII7

$(x) = y = 0{,}033 + 0{,}240x + 2{,}627x^2 - 2{,}126x^3$

$f'(x) = \frac{-3189x^2}{500} + \frac{2627x}{500} + 0{,}24$

$\int_0^1 \frac{-3189x^2}{500} + \frac{2627x}{500} + 0{,}24dx = 0{,}741$

F(GII6)

F(GII6)=GII1

$$(x) = y = 0{,}117 + 1{,}390x - 0{,}414x^2 - 0{,}279x^3$$

$$f'(x) = \frac{- - 837x^2}{1000} - \frac{207x}{250} + 1{,}39$$

$$\int_0^1 \frac{-837x^2}{1000} - \frac{207x}{250} + 1{,}39dx = 0{,}697$$

F(GII6)=GII2

$$(x) = y = 0{,}143 + 1{,}916x - 2{,}047x^2 + 0{,}866x^3$$

$$f'(x) = \frac{1299x^2}{500} - \frac{2047x}{500} + 1{,}916$$

$$\int_0^1 \frac{1299x^2}{500} - \frac{2047x}{500} + 1{,}916dx = 0{,}735$$

F(GII6)=GII3

$$(x) = y = 0{,}164 - 0{,}158x + 3{,}630x^2 - 2{,}874x^3$$

$$f'(x) = \frac{-4311x^2}{500} + \frac{363x}{50} - 0{,}158$$

$$\int_0^1 \frac{-4311x^2}{500} + \frac{363x}{50} - 0{,}158dx = 0{,}598$$

F(GII6)=GII4

$$(x) = y = 0{,}130 + 1{,}841x - 3{,}700x^2 + 2{,}582x^3$$

$$f'(x) = \frac{3873x^2}{500} - \frac{37x}{5} + 1{,}841$$

$$\int_0^1 \frac{3873x^2}{500} - \frac{37x}{5} + 1{,}841dx = 0{,}723$$

F(GII6)=GII5

$$f(x) = y = 0{,}089 + 1{,}370x - 1{,}372x^2 + 0{,}850x^3$$

$$f'(x) = \frac{51x^2}{20} - \frac{343x}{125} + 1{,}37$$

$$\int_0^1 \frac{51x^2}{20} - \frac{343x}{125} + 1{,}37dx = 0{,}848$$

F(GII6)=GII7

$$f(x) = y = 0{,}085 + 0{,}570x + 1{,}417x^2 - 1{,}214x^3$$

$$f'(x) = \frac{-1821x^2}{500} + \frac{1417x}{500} + 0{,}57$$

$$\int_0^1 \frac{-1821x^2}{500} + \frac{1417x}{500} + 0{,}57dx = 0{,}773$$

F(GII7)

F(GII7)=GII1

$$f(x) = y = 0{,}304 + -1{,}020x + 4{,}578x^2 - 3{,}098x^3$$

$$f'(x) = \frac{-4647x^2}{500} + \frac{2289x}{250} - 1{,}02$$

$$\int_0^1 \frac{-4647x^2}{500} + \frac{2289x}{250} - 1{,}02dx = 0{,}460$$

F(GII7)=GII2

$$f(x) = y = 0{,}137 + 1{,}858x - 2{,}465x^2 + 1{,}418x^3$$

$$f'(x) = \frac{2127x^2}{500} - \frac{493x}{100} + 1{,}858$$

$$\int_0^1 \frac{2127x^2}{500} - \frac{493x}{100} + 1{,}858dx = 0{,}811$$

F(GII7)=GII3

$$f(x) = y = 0{,}0800 + 0{,}450x + 1{,}953x^2 - 1{,}635x^3$$

$$f'(x) = \frac{-981x^2}{200} + \frac{1953x}{500} + 0{,}45$$

$$\int_0^1 \frac{-981x^2}{200} + \frac{1953x}{500} + 0{,}45dx = 0{,}768$$

F(GII7)=GII4

$$f(x) = y = 0{,}326 - 1{,}144x + 4{,}083x^2 - 2{,}700x^3$$

$$f'(x) = \frac{-81x^2}{10} + \frac{4083x}{500} - 1{,}144$$

$$\int_0^1 \frac{-81x^2}{10} + \frac{4083x}{500} - 1{,}144dx = 0{,}239$$

F(GII7)=GII5

$$f(x) = y = 0{,}091 + 0{,}915x + 0{,}288x^2 - 0{,}517x^3$$

$$f'(x) = \frac{-1551x^2}{1000} + \frac{72x}{125} + 0{,}915$$

$$\int_0^1 \frac{-1551x^2}{1000} + \frac{72x}{125} + 0{,}915dx = 0{,}686$$

F(GII7)=GII6

$$f(x) = y = 0{,}071 + 0{,}202x + 1{,}661x^2 - 1{,}060x^3$$

$$f'(x) = \frac{-159x^2}{50} + \frac{1661x}{500} + 0{,}202$$

$$\int_0^1 \frac{-159x^2}{50} + \frac{1661x}{500} + 0{,}202dx = 0{,}803$$